

Paper Reference 9FM0/4D
Pearson Edexcel Level 3 GCE

Further Mathematics
Advanced
PAPER 4D: Decision Mathematics 2

Monday 26 June 2023 – Afternoon

Time: 1 hour 30 minutes

YOU MUST HAVE
Mathematical Formulae and Statistical Tables (Green),
calculator

YOU WILL BE GIVEN
Diagram Booklet
Answer Booklet

Q74086A

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Do not return the Question Paper with the Answer Booklet.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the spaces provided in the Answer Booklet or on the separate diagrams in the Diagram Booklet – there may be more space than you need.

Do NOT write on this Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

Turn over

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

**There are 8 questions in this Question Paper.
The total mark for this paper is 75.**

**The marks for EACH question are shown in brackets
– use this as a guide as to how much time to spend on
each question.**

**There are two copies of each diagram in case you need
them.**

ADVICE

**Read each question carefully before you start to answer
it.**

Try to answer every question.

Check your answers if you have time at the end.

1. Refer to Diagram 1 in the Diagram Booklet.

It shows a capacitated, directed network of pipes.

The number on each arc represents the capacity of that pipe.

The numbers in circles represent a feasible flow from **S to **T****

(a) State the value of the feasible flow.

(1 mark)

(b) Find the capacity of cut C_1 and the capacity of cut C_2

(2 marks)

(c) By inspection, find a flow–augmenting route to increase the flow by two units.

You must state your route.

(1 mark)

(continued on the next page)

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1. continued.

(d) Prove that, once the flow–augmenting route found in (c) has been applied, the flow is now maximal.

(3 marks)

(Total for Question 1 is 7 marks)

2. An outdoor theatre is holding a summer gala performance.

The theatre owner must decide whether to take out insurance against rain for this performance.

The theatre owner estimates that

- on a fine day, the total profit will be £15 000
- on a wet day, the total loss will be £20 000

Insurance against rain costs £2 000.

If the performance must be cancelled due to rain, then the theatre owner will receive £16 000 from the insurer.

If the performance is not cancelled due to rain, then the theatre owner will receive nothing from the insurer.

The probability of rain on the day of the gala performance is 0.2

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Turn over

2. continued.

Draw a decision tree and hence determine whether the theatre owner should take out the insurance against rain for this performance.

(5 marks)

(Total for Question 2 is 5 marks)

3. Refer to Diagram 2 in the Diagram Booklet.

It is a table showing the stock held at each supply point and the stock required at each demand point in a standard transportation problem.

The table also shows the cost, in pounds, of transporting the stock from each supply point to each demand point.

The problem is partially described by the linear programming formulation below.

Let x_{ij} be the number of units transported from i to j

where

$$i \in \{A, B, C, D\}$$

$$j \in \{Q, R, S\} \text{ and } x_{ij} \geq 0$$

(continued on the next page)

3. continued.

Minimise

$$\begin{aligned}
 P = & 23x_{AQ} + 18x_{AR} + 12x_{AS} + 8x_{BQ} \\
 & + 10x_{BR} + 14x_{BS} + 11x_{CQ} + 14x_{CR} \\
 & + 21x_{CS} + 19x_{DQ} + 15x_{DR} + 11x_{DS}
 \end{aligned}$$

- (a) Write down, as inequalities, the constraints of the linear program.
(2 marks)
- (b) Use the north–west corner method to obtain an initial solution to this transportation problem.
(1 mark)
- (c) Taking **AS** as the entering cell, use the stepping–stone method to find an improved solution.
Make your route clear.
(2 marks)

(continued on the next page)

Turn over

3. continued.

(d) Perform one further iteration of the stepping–stone method to obtain an improved solution.

You must make your method clear by showing the route and the

- shadow costs**
 - improvement indices**
 - entering cell and exiting cell**
- (4 marks)**

(Total for Question 3 is 9 marks)

4. Refer to Diagram 3 in the Diagram Booklet.

Four students, **A**, **B**, **C** and **D**, are to be allocated to four rounds, **1**, **2**, **3** and **4**, in a competition.

Each student is to take part in exactly one round and no two students may play in the same round.

Each student has been given an estimated score for each round.

The estimated scores for each student are shown in Diagram 3

(continued on the next page)

4. continued.

- (a) Reducing rows first, use the Hungarian algorithm to obtain an allocation that maximises the total estimated score.**

You must make your method clear and show the table after each stage.

(7 marks)

- (b) Find this total estimated score.**

(1 mark)

(Total for Question 4 is 8 marks)

5. A sequence $\{u_n\}$, where $n \geq 0$, satisfies the second order recurrence relation

$$u_{n+2} = \frac{1}{2} (u_{n+1} + u_n) + 3 \text{ where}$$

$$u_0 = 15 \quad u_1 = 20$$

- (a) By considering the sequence $\{v_n\}$, where $u_n = v_n + 2n$ for $n \geq 0$, determine an expression for u_n as a function of n
(7 marks)

- (b) Describe the long-term behaviour of u_n
(1 mark)

(Total for Question 5 is 8 marks)

6. Refer to Diagram 4, Diagram 5, Diagram 6 and Diagram 7 in the Diagram Booklet.

Polly is a motivational speaker who is planning her engagements for the next four weeks.

Polly will

- visit four different countries in these four weeks
- visit just one country each week
- leave from her home, **S**, and return there only after visiting the four countries
- travel directly from one country to the next

Polly wishes to determine a schedule of four countries to visit.

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6. continued.

Diagram 4 shows the countries Polly could visit each week.

Diagram 5 shows the speaker fee, in £100s, Polly would expect to earn in each country.

Diagram 6 shows the cost, in £100s, of travelling between the countries.

Polly's expected income is the value of the speaker fee minus the cost of travel.

She wants to find a schedule that maximises her total expected income for the four weeks.

Use dynamic programming to determine the optimal schedule.

Complete Diagram 7 in the Diagram Booklet and state the maximum expected income.

(13 marks)

(Total for Question 6 is 13 marks)

Turn over

7. Martina decides to open a bank account to help her to save for a holiday.

Each month she puts £ k into the account and allows herself to spend one quarter of what was in the account at the end of the previous month.

Let u_n (where $n \geq 1$) represent the amount in the account at the end of month n

Martina has £ k in the account at the end of the first month.

- (a) By setting up a first order recurrence relation for u_{n+1} in terms of u_n , determine an expression for u_n in terms of n and k
(6 marks)

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7. continued.

At the end of the 8th month, Martina needs to have at least £1750 in the account to pay for her holiday.

(b) Determine, to the nearest penny, the minimum amount of money that Martina should put into the account each month.

(2 marks)

(Total for Question 7 is 8 marks)

8. Refer to Diagram 8 in the Diagram Booklet.

A two-person zero-sum game is represented by the pay-off matrix for player **A** shown in Diagram 8

(a) Verify that there is no stable solution to this game.

(2 marks)

(b) Explain why player **A** should never play option **T**

You must make your reasoning clear.

(2 marks)

(continued on the next page)

8. continued.

Player **A** intends to make a random choice between options **Q**, **R** and **S**, choosing option **Q** with probability p_1 , option **R** with probability p_2 and option **S** with probability p_3

Player **A** wants to calculate the optimal values of p_1 , p_2 and p_3 using the Simplex algorithm.

(c) (i) Formulate the game as a linear programming problem for player **A**
You should write the constraints as equations.

(ii) Write down an initial Simplex tableau for this linear programming problem, making your variables clear.

(7 marks)

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Turn over

8. continued.

The linear programming problem is solved using the Simplex algorithm.

The optimal value of p_1 is $\frac{6}{11}$ and the optimal value of p_2 is 0

(d) Find the best strategy for player B, defining any variables you use.

(6 marks)

(Total for Question 8 is 17 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
